CE 4780 Hurricane Engineering II

Section on Flooding Protection: Earth Retaining Structures and Slope Stability

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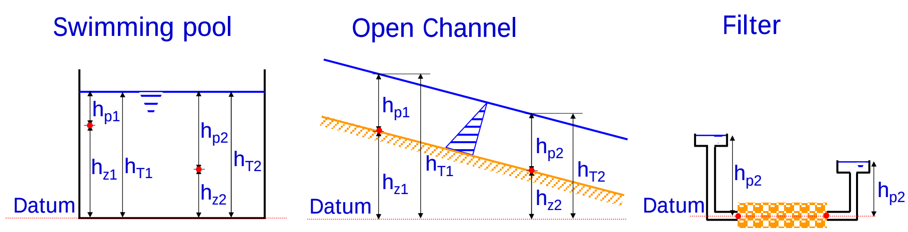
- Three weeks of classes

Seepage Analysis

◆Introduction

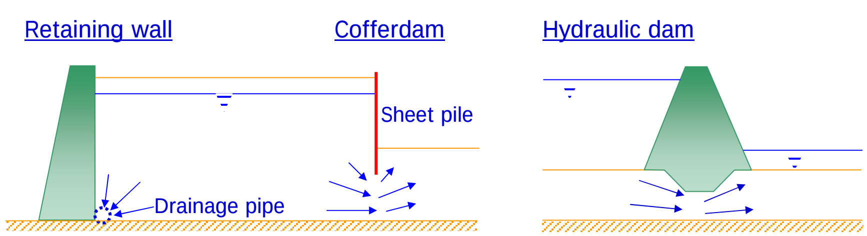


◆ Flow is governed by the total head!!

Seepage Analysis

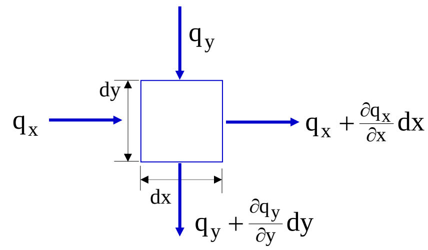
◆Objectives

- To obtain pore pressure (stability analysis)
- To calculate flow
- To verify piping conditions



Laplace's Equation

- Elemental Cube:
  - Saturation S=100 %
  - Void ratio e= constant
  - Laminar flow



- Continuity: $q_{in} = q_{out}$

$$q_x + q_y - \left(q_x + \frac{\partial q_x}{\partial x} dx + q_y + \frac{\partial q_y}{\partial y} dy \right) = 0$$

$$\frac{\partial q_x}{\partial x} dx + \frac{\partial q_y}{\partial y} dy = 0$$

Laplace's Equation

- Continuity: $\frac{\partial q_x}{\partial x} dx + \frac{\partial q_y}{\partial y} dy = 0$
- Darcy's law: $q_x = k_x \cdot i \cdot A = k_x \cdot \frac{\partial h_T}{\partial x} \cdot dy \cdot 1$
- Replacing: $0 = k_x \cdot \frac{\partial^2 h_T}{\partial x^2} dx \cdot dy \cdot 1 + k_y \cdot \frac{\partial^2 h_T}{\partial y^2} dy \cdot dx \cdot 1$

$$0 = k_x \cdot \frac{\partial^2 h_T}{\partial x^2} + k_y \cdot \frac{\partial^2 h_T}{\partial y^2}$$

- if $k_x = k_y$ (isotropy): $0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2}$ Laplace's Equation!

Laplace's Equation

- Typical cases
  - 1 Dimensional: $0 = \frac{\partial^2 h_T}{\partial x^2}$

    $\text{constant} = \frac{\partial h_T}{\partial x} = i$

    linear variation!! $h_T = a + b \cdot x$
  - 2-Dimensional: $0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2}$
  - 3-Dimensional: $0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2} + \frac{\partial^2 h_T}{\partial z^2}$

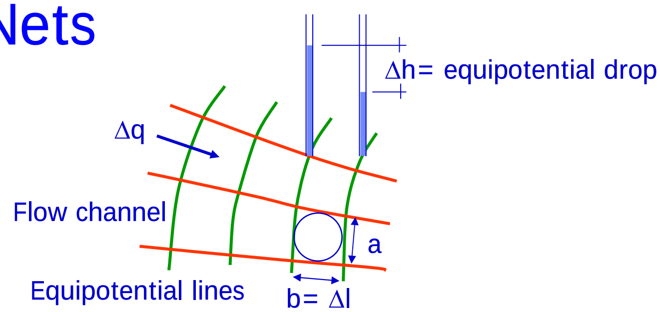
Laplace's Equation Solutions

- Exact solutions (for simple B.C.'s)
- Physical models (scaling problems)
- Graphical solutions: flow nets
- Analogies: heat flow and electrical flow
- Numerical solutions: finite differences
- Approximate solutions: method of fragments

Flow Nets

- The procedure consists on drawing a set of perpendicular lines: equi-potentials and flow lines.
- These set of lines are the solution to the Laplace's equation.
- It is an iterative (and tedious!) process.
- Identify boundaries:
  - First and last equi-potentials
  - First and last flow lines

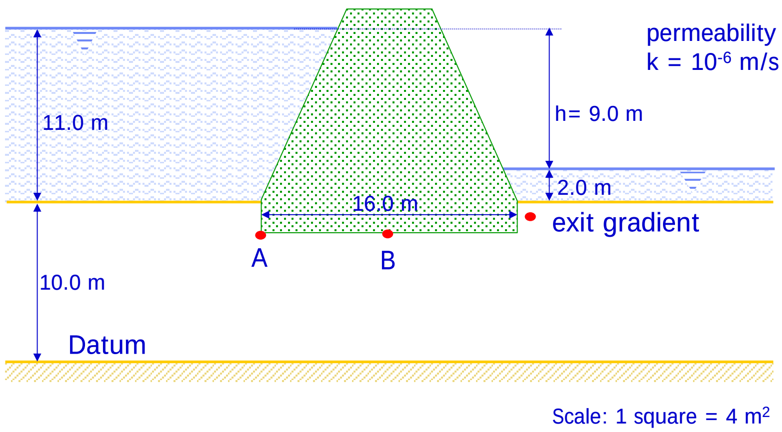
Flow Nets



- gradient: $i = \frac{\Delta h}{\Delta l} = \frac{\Delta h}{b} = \frac{h/N_e}{b}$
- flow per channel: $\Delta q = k \cdot \frac{\Delta h}{\Delta l} \cdot A = k \cdot \frac{h/N_e}{b} \cdot A$
- total flow: $q = \Delta q \cdot N_f = k \cdot h \cdot \frac{a}{b} \cdot \frac{N_f}{N_e}$

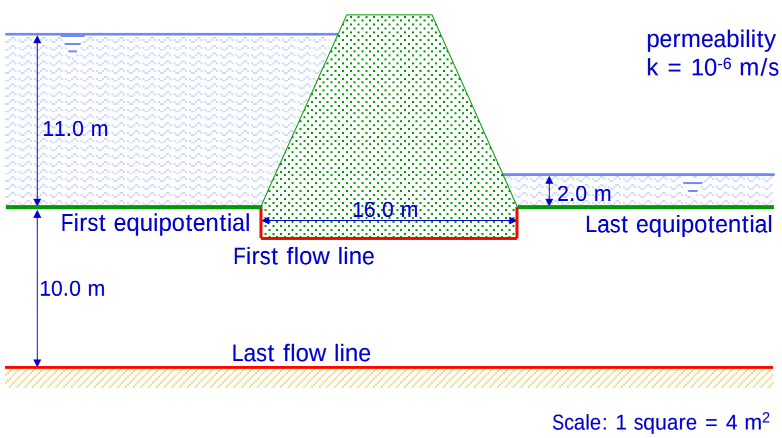
Flow Nets– Confined Flow

- Example



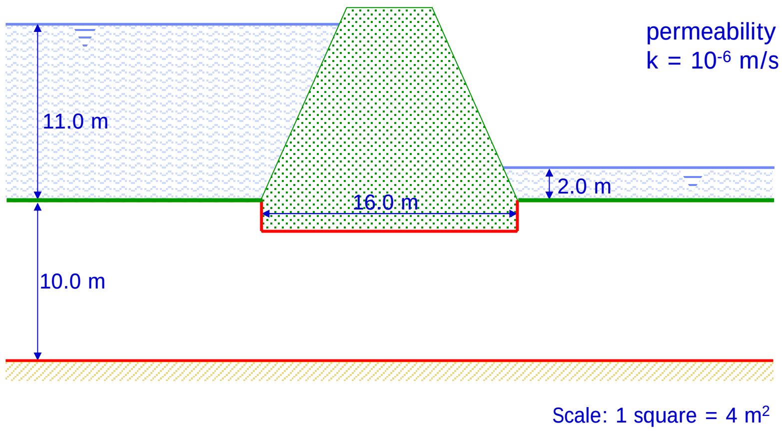
Flow Nets – Confined Flow

- Example



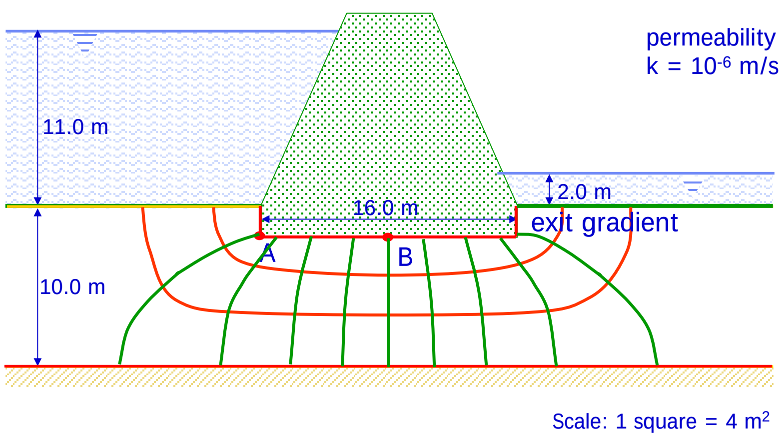
Flow Nets – Confined Flow

- Example



Flow Nets – Confined Flow

- Example



Flow Nets – Confined Flow

- Example

Seepage loss under the dam:

$$q = k \cdot \Delta h \cdot \left(\frac{N_f}{N_e} \right) = 10^{-6} \frac{m}{s} \cdot 9\,m \cdot \frac{3}{10} = 2.7 \cdot 10^{-6} \frac{m^3}{s \cdot m}$$

Exit gradient:

$$i_e = \frac{\Delta h}{\Delta l} = \frac{h/N_e}{\Delta l} = \frac{0.9\,m}{2\,m} = 0.45$$

$$i_{crit} = 1 \therefore \Delta h \cdot \gamma_w = \Delta l \cdot (\gamma_{sat} - \gamma_w) \Rightarrow \text{piping!!}$$

Flow Nets

- Piping

(from McCarthy 1998)

Flow Nets

Example

Total head at points A and B:

$$h_{TA} = h_{To} - h \cdot \frac{N_A}{N_f} = 21m - 9m \cdot \frac{1}{10} = 20.1m$$

$$h_{TB} = h_{To} - h \cdot \frac{N_B}{N_f} = 21m - 9m \cdot \frac{5}{10} = 16.5m$$

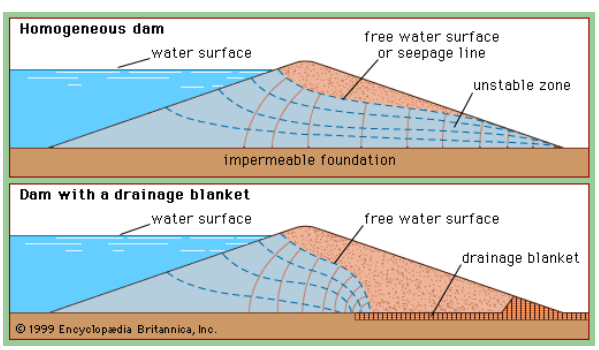
Pressure head at points A and B:

$$h_{PA} = h_{TA} - h_{zA} = 20.1m - 8m = 12.1m \Rightarrow \approx 121kPa$$

$$h_{PB} = h_{TB} - h_{zB} = 16.5m - 8m = 8.5m \Rightarrow \approx 85kPa$$

Flow Nets – Unconfined Flow

Example:



Flow Nets - Unconfined Flow

- Line of Seepage

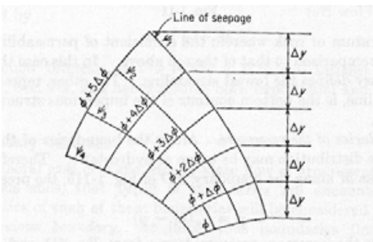


Fig. 1-12

(from Harr 1990)

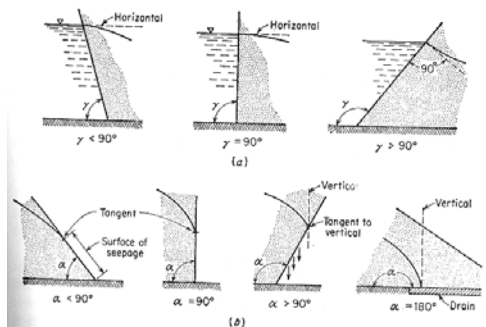


Fig. 1-13. (*After A. Casagrande* [14].)

Electrical Flow Analogy

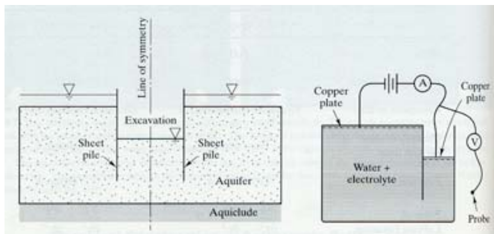
- Electric conduction
  - Ohm's law: $V = I \cdot R \therefore I = \frac{V}{R}$
  - Resistance: $R = \rho \cdot \frac{L}{A}$

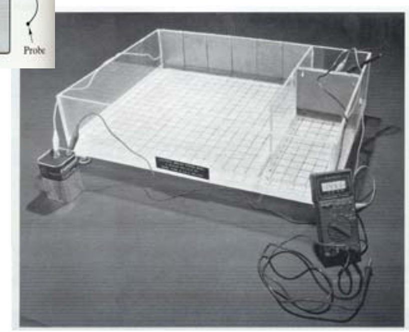
$$I = \frac{1}{\rho} \cdot \frac{V}{L} \cdot A$$

$$I = \frac{1}{\rho} \cdot i_e \cdot A$$

  - Darcy's law: $q = k \cdot i_h \cdot A$

Electrical Flow Analogy

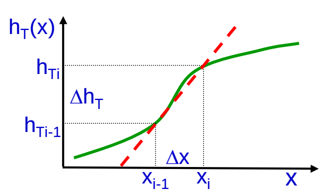




(from Coduto 1999)

Finite Differences Method

- Laplace's equation: $0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2}$



- Finite differences: $\frac{\partial h_T}{\partial x} \approx \frac{\Delta h_T}{\Delta x} = \frac{1}{\Delta x} \cdot \left(h_{T_i} - h_{T_{i-1}}\right)$

  (Taylor's series expansions) $\frac{\partial^2 h_T}{\partial x^2} \approx \frac{1}{\Delta x^2} \cdot \left(h_{T_{i+1}} - 2 \cdot h_{T_i} + h_{T_{i-1}}\right)$

Finite Differences Method

- Laplace's equation in finite differences:

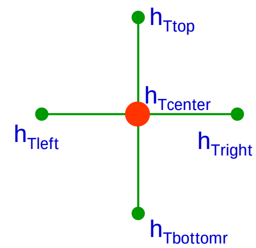
$$0 = \frac{1}{\Delta x^2}\cdot\left(h_{T_{i+1,k}} - 2\cdot h_{T_{i,k}} + h_{T_{i-1,k}}\right) + \frac{1}{\Delta y^2}\cdot\left(h_{T_{i,k+1}} - 2\cdot h_{T_{i,k}} + h_{T_{i,k-1}}\right)$$

- Solving for $h_{Ti,k}$ (if $\Delta x = \Delta y$):

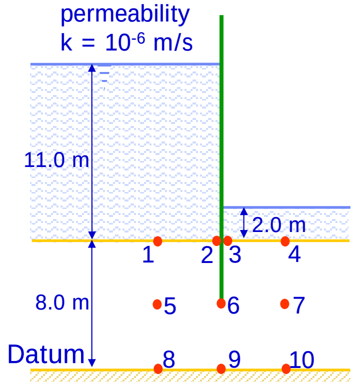
$$h_{T_{i,k}} = \frac{1}{4}\cdot\left(h_{T_{i+1,k}} + h_{T_{i-1,k}} + h_{T_{i,k+1}} + h_{T_{i,k-1}}\right)$$

or

$$h_{T_{center}} = \frac{1}{4}\cdot\left(h_{T_{left}} + h_{T_{right}} + h_{T_{bottom}} + h_{T_{top}}\right)$$



Finite Differences Method

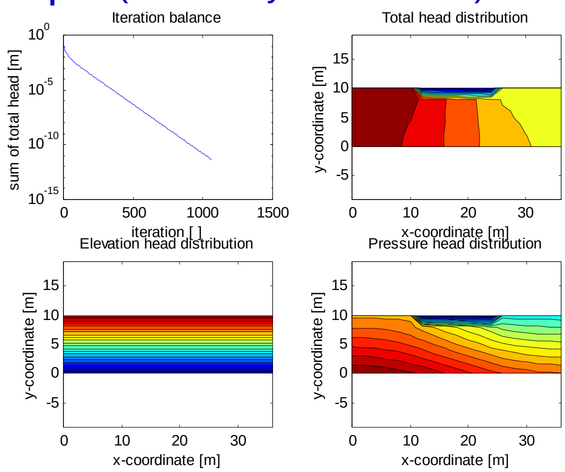


Scale: 1 square = 4 m^2

Number	Head	Head	Top	Botton	Left	Right	Head
1	19	8	1	1	1	1	19
2	19	8	2	2	2	2	19
3	10	8	3	3	3	3	10
4	10	8	4	4	4	4	10
5	0	4	1	8	6	6	4.75
6	0	4	2	9	5	7	4.75
7	0	4	4	10	6	6	2.5
8	0	0	5	5	9	9	0
9	0	0	6	6	8	10	0
10	0	0	7	7	9	9	0

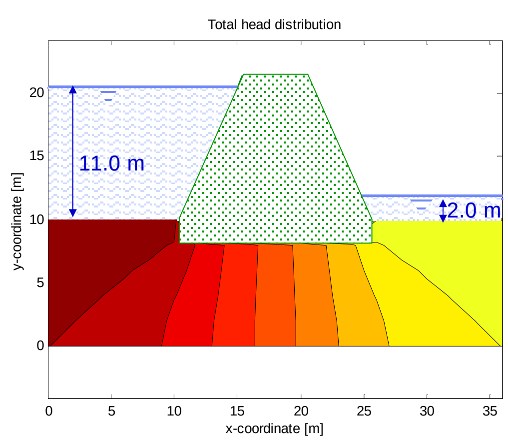
Finite Differences Method

- Example (solution by Wes Sherrod)



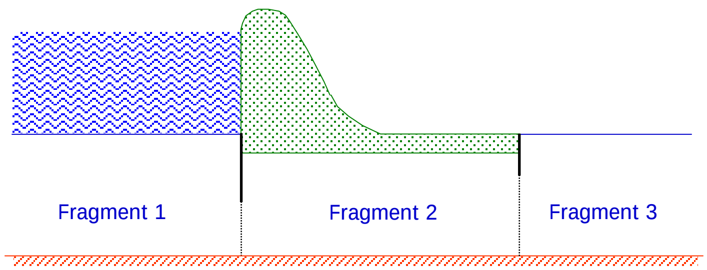
Finite Differences Method

- Example (solution by Wes Sherrod - GTREP)



Method of Fragments

- The fundamental assumption in the method of fragments is that the problem can be divided in segments with flow characterized by vertical equipotential lines (Harr 1990).



(after Holtz and Kovacks 1981)

Method of Fragments

- The flow through a fragment m is computed as (Harr 1990)

$$q = k \cdot \frac{\Delta h_{tm}}{\Phi_m}$$

Where k is the hydraulic conductivity, Δh_{tm} is the total head drop in segment m, and Φ_m is the dimensionless form factor

Method of Fragments

- Because the discharge in each segment must be the same, then

$$Q = q = k \cdot \frac{\Delta h_{t1}}{\Phi_1} = k \cdot \frac{\Delta h_{t2}}{\Phi_2} = \cdots = k \cdot \frac{\Delta h_{tn}}{\Phi_n}$$

$$Q = k \cdot \frac{\sum_i \Delta h_{ti}}{\sum_i \Phi_i} = k \cdot \frac{\Delta H_t}{\sum_i \Phi_i}$$

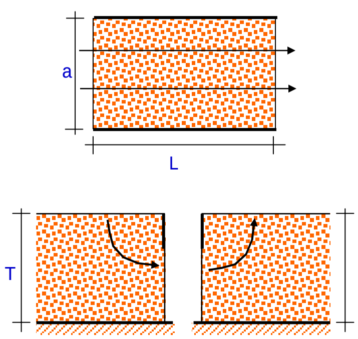
and the head loss in each segment is:

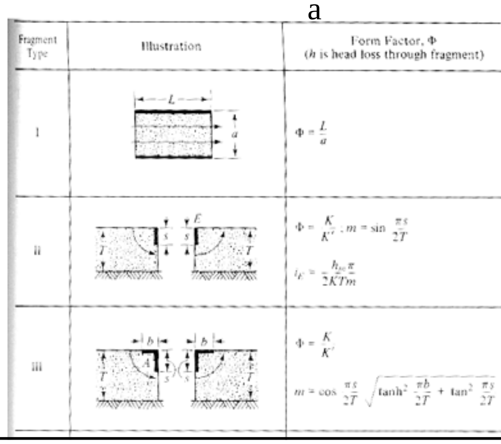
$$\Delta h_{ti} = \Delta H_t \cdot \frac{\Phi_i}{\sum_i \Phi_i}$$

Method of Fragments

- Fragment types and form factors

Fragment type I: $\Phi = \frac{L}{a}$

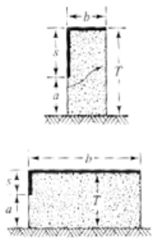
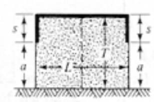




Fragment Type	Illustration	Form Factor, Φ (*h* is head loss through fragment)
I		$\Phi = \frac{L}{a}$
II		$\Phi = \frac{K}{K'}$; $m = \sin\frac{\pi s}{2T}$ $i_E = \frac{h_m \pi}{2KTm}$
III		$\Phi = \frac{K}{K'}$ $m = \cos\frac{\pi s}{2T}\sqrt{\tanh^2\frac{\pi b}{2T} + \tan^2\frac{\pi s}{2T}}$

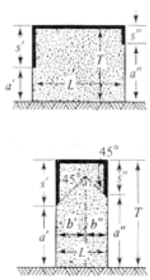
Method of Fragments

- Fragment types and form factors

IV		*Exact solution:* $\frac{\Lambda}{\Lambda'} = \frac{T}{b}$: modulus = λ $\Phi = \frac{K'(m)}{K(m)}$; $m = \lambda \, \mathrm{sn}\left(\frac{a}{T}\Lambda, \lambda\right)$ *Approximate solution:* $s \geq b$: $\Phi = \ln\left(1 + \frac{b}{a}\right)$ $b \geq s$: $\Phi = \ln\left(1 + \frac{s}{a}\right) + \frac{b-s}{T}$
V		$L \leq 2s$: $\Phi = 2\ln\left(1 + \frac{L}{2a}\right)$ $L \geq 2s$: $\Phi = 2\ln\left(1 + \frac{s}{a}\right) + \left(\frac{L-2s}{T}\right)$

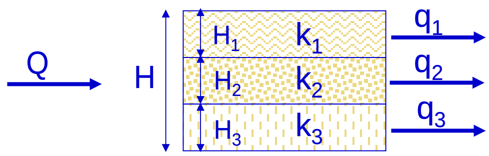
Method of Fragments

- Fragment types and form factors

VI		$L > s' + s''$: $\Phi = \ln\left[\left(1 + \frac{s'}{a'}\right)\left(1 + \frac{s''}{a''}\right)\right] + \frac{L - (s' + s'')}{T}$ $L = s' + s''$: $\Phi = \ln\left[\left(1 + \frac{s'}{a'}\right)\left(1 + \frac{s''}{a''}\right)\right]$ $L < s' + s''$ $\Phi = \ln\left[\left(1 + \frac{b'}{a'}\right)\left(1 + \frac{b''}{a''}\right)\right]$ where $b' = \frac{L + (s' - s'')}{2}$ $b'' = \frac{L - (s' - s'')}{2}$

Flow Through Heterogeneous Media

- Effective Permeability in Stratified Soils

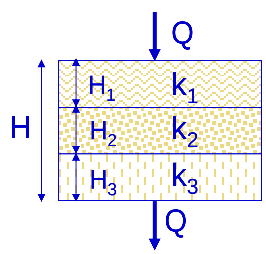


$$\left.\begin{aligned} q_1 &= k_1 \cdot i_h \cdot H_1 \cdot 1 \\ q_2 &= k_2 \cdot i_h \cdot H_2 \cdot 1 \\ q_3 &= k_3 \cdot i_h \cdot H_3 \cdot 1 \end{aligned}\right\} \equiv Q = k_{eq-horiz} \cdot i_h \cdot H \cdot 1$$

$$k_{eq-horiz} = \frac{1}{H} \cdot \left(k_1 \cdot H_1 + k_2 \cdot H_2 + k_3 \cdot H_3\right)$$

Flow Through Heterogeneous Media

- Effective Permeability in Stratified Soils



$$Q = k_{eq-vert} \cdot i_h \cdot A$$

$$= k_1 \cdot i_{h1} \cdot A = k_2 \cdot i_{h2} \cdot A = k_3 \cdot i_{h3} \cdot A$$

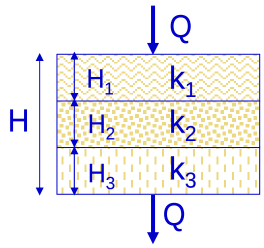
$$i_h \cdot H = i_{h1} \cdot H_1 + i_{h2} \cdot H_2 + i_{h3} \cdot H_3$$

$$\frac{Q}{k_{eq-vert} \cdot A} \cdot H = \frac{Q}{k_1 \cdot A} \cdot H_1 + \frac{Q}{k_2 \cdot A} \cdot H_2 + \frac{Q}{k_3 \cdot A} \cdot H_3$$

$$k_{eq-vert} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3}}$$

Flow Through Heterogeneous Media

- Effective Permeability in Stratified Soils



$$Q = k_{eq-vert} \cdot i_h \cdot A$$

$$= k_1 \cdot i_{h1} \cdot A = k_2 \cdot i_{h2} \cdot A = k_3 \cdot i_{h3} \cdot A$$

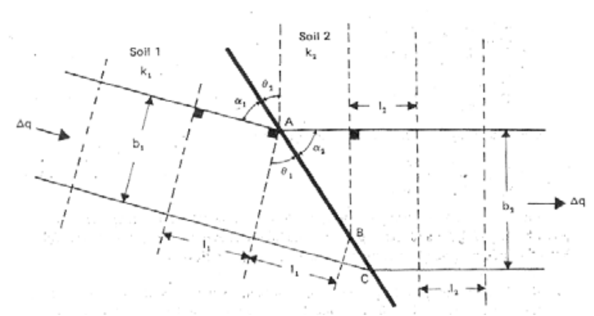
$$i_h \cdot H = i_{h1} \cdot H_1 + i_{h2} \cdot H_2 + i_{h3} \cdot H_3$$

$$\frac{Q}{k_{eq-vert} \cdot A} \cdot H = \frac{Q}{k_1 \cdot A} \cdot H_1 + \frac{Q}{k_2 \cdot A} \cdot H_2 + \frac{Q}{k_3 \cdot A} \cdot H_3$$

$$k_{eq-vert} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3}}$$

Flow Through Heterogeneous Media

- Zones of different hydraulic conductivities



(Das 1983)

$$\Delta q = k_1 \cdot \frac{\Delta h}{l_1} \cdot (b_1 \cdot 1) = k_2 \cdot \frac{\Delta h}{l_2} \cdot (b_2 \cdot 1) \qquad \frac{k_1}{k_2} = \frac{b_2}{l_2} \cdot \frac{l_1}{b_1}$$

Flow Through Heterogeneous Media

- Zones of different hydraulic conductivities

$$l_1 = AB \cdot \sin(\theta_1) = AB \cdot \cos(\alpha_1)$$
$$l_2 = AB \cdot \sin(\theta_2) = AB \cdot \cos(\alpha_2)$$
$$b_1 = AC \cdot \cos(\theta_1) = AC \cdot \sin(\alpha_1)$$
$$b_2 = AC \cdot \cos(\theta_2) = AC \cdot \sin(\alpha_2)$$

then

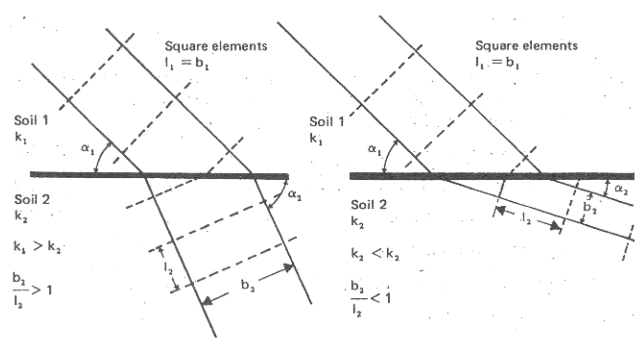
$$\frac{b_1}{l_1} = \frac{\cos(\theta_1)}{\sin(\theta_1)} = \frac{\sin(\alpha_1)}{\cos(\alpha_1)} = \frac{1}{\tan(\theta_1)} = \tan(\alpha_1)$$
$$\frac{b_2}{l_2} = \frac{\cos(\theta_2)}{\sin(\theta_2)} = \frac{\sin(\alpha_2)}{\cos(\alpha_2)} = \frac{1}{\tan(\theta_2)} = \tan(\alpha_2)$$

Flow Through Heterogeneous Media

- Zones of different hydraulic conductivities

$$\frac{k_1}{k_2} = \frac{\tan(\theta_1)}{\tan(\theta_2)} = \frac{\tan(\alpha_2)}{\tan(\alpha_1)}$$

Examples:

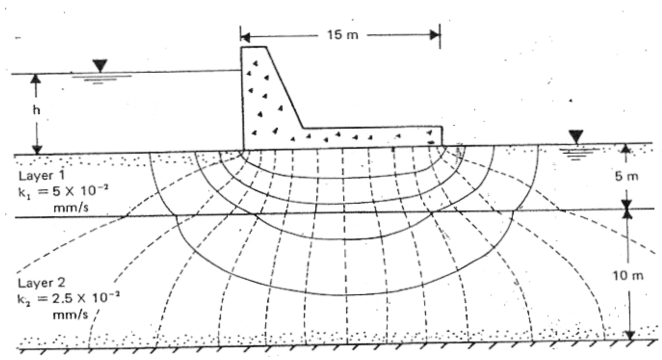


(Das 1983)

Flow Through Heterogeneous Media

- Zones of different hydraulic conductivities

Flow Net Example:



(Das 1983)

Flow Through Anisotropic Media

- Anisotropic Media $k_x \neq k_y$

Laplace's equation: $0 = k_x \cdot \frac{\partial^2 h_T}{\partial x^2} + k_y \cdot \frac{\partial^2 h_T}{\partial y^2}$

The equation may be easily solved by transforming coordinates from The (x,y) space to the (x',y) space

where: $x' = \alpha \cdot x = \sqrt{\frac{k_y}{k_x}} \cdot x$

and: $x = \frac{x'}{\alpha} \Rightarrow \partial x = \partial\left(\frac{x'}{\alpha}\right)$

$$\frac{\partial h_T}{\partial x} = \alpha \cdot \frac{\partial h_T}{\partial x'} \Rightarrow \frac{\partial^2 h_T}{\partial x^2} = \alpha^2 \cdot \frac{\partial^2 h_T}{\partial x'^2}$$

Flow Through Anisotropic Media

- Anisotropic Media $k_x \neq k_y$

Replacing into Laplace's equation:

$$0 = k_x \cdot \alpha^2 \cdot \frac{\partial^2 h_T}{\partial x'^2} + k_y \cdot \frac{\partial^2 h_T}{\partial y^2}$$

$$= k_x \cdot \left(\frac{k_y}{k_x}\right) \cdot \frac{\partial^2 h_T}{\partial x'^2} + k_y \cdot \frac{\partial^2 h_T}{\partial y^2}$$

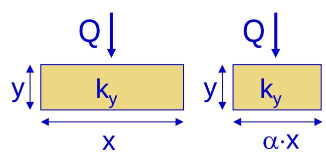
And that is:

$$0 = \frac{\partial^2 h_T}{\partial x'^2} + \frac{\partial^2 h_T}{\partial y^2}$$

The problem may then be solved using flow nets by redrawing the cross-section using $x' = x \cdot \sqrt{(k_y/k_x)}$...

Flow Through Anisotropic Media

- Anisotropic Media $k_x \neq k_y$



The flow in an anisotropic media:

$$Q = \alpha \cdot x \cdot k_{eq} \cdot \frac{\partial h_T}{\partial y} = x \cdot k_y \cdot \frac{\partial h_T}{\partial y}$$

$$\alpha \cdot k_{eq} = k_y \Rightarrow \sqrt{\frac{k_y}{k_y}} \cdot k_{eq} = k_y$$

Transformed permeability:

$$\boxed{k_{eq} = \sqrt{k_x \cdot k_y}}$$

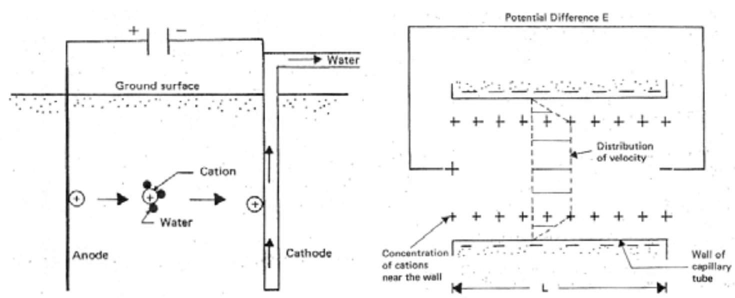
Coupled Flow

- Coupled Phenomena

It is justify by:

  - Le Chaterlier's Principle & Energy Conservation

- Example: Electrosmosis



(Das 1983)

Seepage Control - Filters

- Seepage: Cut-off walls
  Impervious blankets

- Erosion and piping: Filters

Requirements

Piping:	D_{15}(filter) =< 5 D_{85} (soil)
Permeability:	D_{15}(filter) >= 5 D_{15} (soil)
Uniformity:	D_{50}(filter) =< 25 D_{50} (soil)

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