CE 4780 Hurricane Engineering II

Section on Flooding Protection: Earth Retaining Structures and Slope Stability

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- Three weeks of classes
Seepage Analysis

Introduction

Flow is governed by the total head!!

Objectives

- To obtain pore pressure (stability analysis)
- To calculate flow
- To verify piping conditions
Laplace’s Equation

Elemental Cube:
- Saturation $S=100\%$
- Void ratio $e=\text{constant}$
- Laminar flow

Continuity: $q_{\text{in}} = q_{\text{out}}$

\[
q_x + q_y - \left( q_x + \frac{\partial q_x}{\partial x} dx + q_y + \frac{\partial q_y}{\partial y} dy \right) = 0
\]

\[
\frac{\partial q_x}{\partial x} dx + \frac{\partial q_y}{\partial y} dy = 0
\]

Laplace’s Equation

Continuity: $\frac{\partial q_x}{\partial x} dx + \frac{\partial q_y}{\partial y} dy = 0$

Darcy’s law: $q_x = k_x \cdot i \cdot A = k_x \cdot \frac{\partial h_T}{\partial x} \cdot dy \cdot 1$

Replacing:

\[
0 = k_x \cdot \frac{\partial^2 h_T}{\partial x^2} dx \cdot dy \cdot 1 + k_y \cdot \frac{\partial^2 h_T}{\partial y^2} dy \cdot dx \cdot 1
\]

\[
0 = k_x \cdot \frac{\partial^2 h_T}{\partial x^2} + k_y \cdot \frac{\partial^2 h_T}{\partial y^2}
\]

if $k_x = k_y$ (isotropy):

\[
0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2} \quad \text{Laplace’s Equation!}
\]
Laplace’s Equation

Typical cases

- **1 Dimensional:**
  \[ 0 = \frac{\partial^2 h_T}{\partial x^2} \]
  \( \text{constant} \ t = \frac{\partial h_T}{\partial x} = i \)
  \( h_T = a + b \cdot x \)

- **2-Dimensional:**
  \[ 0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2} \]

- **3-Dimensional:**
  \[ 0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2} + \frac{\partial^2 h_T}{\partial z^2} \]

Laplace’s Equation Solutions

- Exact solutions (for simple B.C.’s)
- Physical models (scaling problems)
- Graphical solutions: flow nets
- Analogies: heat flow and electrical flow
- Numerical solutions: finite differences
- Approximate solutions: method of fragments
Flow Nets

- The procedure consists on drawing a set of perpendicular lines: equi-potentials and flow lines.
- These set of lines are the solution to the Laplace’s equation.
- It is an iterative (and tedious!) process.
- Identify boundaries:
  - First and last equi-potentials
  - First and last flow lines

\[ i = \frac{\Delta h}{\Delta l} = \frac{\Delta h}{b} = \frac{h/N_e}{b} \]

\[ \Delta q = k \cdot \frac{\Delta h}{\Delta l} \cdot A = k \cdot \frac{h/N_e}{b} \cdot A \]

\[ q = \Delta q \cdot N_f = k \cdot h \cdot \frac{a}{b} \cdot \frac{N_f}{N_e} \]
Flow Nets– Confined Flow

Example

Datum

11.0 m

2.0 m

10.0 m

16.0 m

permeability
k = $10^{-6}$ m/s

h = 9.0 m

exit gradient

Scale: 1 square = 4 m²
Flow Nets – Confined Flow

Example

permeability
$k = 10^{-6}$ m/s

Scale: 1 square = 4 m$^2$
Flow Nets – Confined Flow

Example

Seepage loss under the dam:

\[ q = k \cdot \Delta h \cdot \left( \frac{N_f}{N_c} \right) = 10^{-6} \frac{m}{s} \cdot 9 \m \cdot \frac{3}{10} = 2.7 \cdot 10^{-6} \frac{m^3}{s \cdot m} \]

Exit gradient:

\[ i_e = \frac{\Delta h}{\Delta l} = \frac{h}{N_c} = \frac{0.9}{2} = 0.45 \]

\[ i_{crit} = 1 : \Delta h \cdot \gamma_w = \Delta l \cdot (\gamma_{sat} - \gamma_w) \Rightarrow \text{piping!!} \]

Flow Nets

Piping

(from McCarthy 1998)
Flow Nets

Example
Total head at points A and B:
\[ h_{TA} = h_{To} - h \cdot \frac{N_A}{N_f} = 21m - 9m \cdot \frac{1}{10} = 20.1m \]
\[ h_{TB} = h_{To} - h \cdot \frac{N_B}{N_f} = 21m - 9m \cdot \frac{5}{10} = 16.5m \]
Pressure head at points A and B:
\[ h_{PA} = h_{TA} - h_{zA} = 20.1m - 8m = 12.1m \Rightarrow \approx 121kPa \]
\[ h_{PB} = h_{TB} - h_{zB} = 16.5m - 8m = 8.5m \Rightarrow \approx 85kPa \]

Flow Nets – Unconfined Flow

Example:
Flow Nets - Unconfined Flow

Line of Seepage

(from Harr 1990)

Electrical Flow Analogy

Electric conduction

- Ohm’s law: \( V = I \cdot R \) \( \therefore I = \frac{V}{R} \)
- Resistance: \( R = \frac{\rho \cdot L}{A} \)
  \( I = \frac{V}{\rho \cdot L} \cdot A \)
  \( I = \frac{1}{\rho} \cdot i_c \cdot A \)
- Darcy’s law: \( q = k \cdot i_h \cdot A \)
Electrical Flow Analogy

(from Coduto 1999)

Finite Differences Method

- **Laplace’s equation:** \[ 0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2} \]

- **Finite differences:** (Taylor’s series expansions)
  \[ \frac{\partial h_T}{\partial x} \approx \frac{\Delta h_T}{\Delta x} = \frac{1}{\Delta x} \cdot (h_{T_i} - h_{T_{i-1}}) \]
  \[ \frac{\partial^2 h_T}{\partial x^2} \approx \frac{1}{\Delta x^2} \cdot (h_{T_{i+1}} - 2 \cdot h_{T_i} + h_{T_{i-1}}) \]
Finite Differences Method

Laplace’s equation in finite differences:

\[ 0 = \frac{1}{\Delta x^2} \left( h_{r_{i+1,k}} - 2 \cdot h_{r_{i,k}} + h_{r_{i-1,k}} \right) + \frac{1}{\Delta y^2} \left( h_{r_{i,k+1}} - 2 \cdot h_{r_{i,k}} + h_{r_{i,k-1}} \right) \]

Solving for \( h_{r_{i,k}} \) (if \( \Delta x = \Delta y \)):

\[ h_{r_{i,k}} = \frac{1}{4} \left( h_{r_{i+1,k}} + h_{r_{i-1,k}} + h_{r_{i,k+1}} + h_{r_{i,k-1}} \right) \]

or

\[ h_{r_{\text{center}}} = \frac{1}{4} \left( h_{r_{\text{left}}} + h_{r_{\text{right}}} + h_{r_{\text{bottom}}} + h_{r_{\text{top}}} \right) \]
Finite Differences Method

**Example (solution by Wes Sherrod)**

- **Iteration balance**
  - x-coordinate [m]
  - y-coordinate [m]

- **Total head distribution**
  - x-coordinate [m]
  - y-coordinate [m]

- **Elevation head distribution**
  - x-coordinate [m]
  - y-coordinate [m]

- **Pressure head distribution**
  - x-coordinate [m]
  - y-coordinate [m]

- **Total head distribution**
  - x-coordinate [m]
  - y-coordinate [m]

- **Elevation head distribution**
  - x-coordinate [m]
  - y-coordinate [m]

- **Pressure head distribution**
  - x-coordinate [m]
  - y-coordinate [m]
Method of Fragments

The fundamental assumption in the method of fragments is that the problem can be divided into segments with flow characterized by vertical equipotential lines (Harr 1990).

\[ q = k \cdot \frac{\Delta h_{tm}}{\Phi_m} \]

Where \( k \) is the hydraulic conductivity, \( \Delta h_{tm} \) is the total head drop in segment \( m \), and \( \Phi_m \) is the dimensionless form factor

(after Holtz and Kovacks 1981)
Method of Fragments

Because the discharge in each segment must be the same, then

\[ Q = q = k \cdot \frac{\Delta h_{11}}{\Phi_1} = k \cdot \frac{\Delta h_{12}}{\Phi_2} = \ldots = k \cdot \frac{\Delta h_{1n}}{\Phi_n} \]

\[ Q = k \cdot \sum_i \frac{\Delta h_{ii}}{\Phi_i} = k \cdot \sum_i \frac{\Delta H_i}{\Phi_i} \]

and the head loss in each segment is:

\[ \Delta h_{ii} = \Delta H_i \cdot \frac{\Phi_i}{\sum_i \Phi_i} \]
Method of Fragments

Fragment types and form factors

<table>
<thead>
<tr>
<th>Fragment Types</th>
<th>Form Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
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- Exact solution:
  - $N = \frac{f}{R}$ (modulus $= \lambda$)
  - $\Phi = \frac{K^\text{sec}}{K^\text{sec} + \omega}$ (where $\omega = \frac{\beta}{\lambda}$)

- Approximate solution:
  - $N > m$
    - $\Phi = \frac{1 + \frac{L}{2}}{2}$
  - $N \leq m$
    - $\Phi = \frac{1 + \frac{L}{2} - \frac{1}{\sqrt{2}}}{2}$

<table>
<thead>
<tr>
<th>V</th>
<th></th>
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- Exact solution:
  - $V = \frac{2L}{1 + \frac{L}{2}}$
  - $L \leq 2L$
    - $\Phi = 2L$ (where $\gamma = \frac{L}{2}$)
  - $L > 2L$
    - $\Phi = 2L$ (where $\gamma = \frac{L}{2}$)

Method of Fragments

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- Exact solution:
  - $V = \frac{1 + \frac{L}{2}}{2}$
  - $\Phi = \ln \left( \frac{1 + \frac{L}{2} + \frac{\gamma}{\lambda}}{\lambda - 1 + \frac{\gamma}{\lambda}} \right)$
  - $L = \frac{\gamma}{\lambda}$
  - $L \leq 2L$
    - $\Phi = \ln \left( \frac{1 + \frac{L}{2} + \frac{\gamma}{\lambda}}{\lambda - 1 + \frac{\gamma}{\lambda}} \right)$
  - $L > 2L$
    - $\Phi = \ln \left( \frac{1 + \frac{L}{2} + \frac{\gamma}{\lambda}}{\lambda - 1 + \frac{\gamma}{\lambda}} \right)$

where
- $\gamma = \frac{L + (\gamma - L)}{2}$
- $\gamma = \frac{L - (\gamma - L)}{2}$
Flow Through Heterogeneous Media

Effective Permeability in Stratified Soils

\[ Q = k_{eq\text{-}horiz} \cdot i_h \cdot H \cdot 1 \]

\[ k_{eq\text{-}horiz} = \frac{1}{H} \left( k_1 \cdot H_1 + k_2 \cdot H_2 + k_3 \cdot H_3 \right) \]

\[ \begin{align*}
q_1 &= k_1 \cdot i_h \cdot H_1 \cdot 1 \\
q_2 &= k_2 \cdot i_h \cdot H_2 \cdot 1 \\
q_3 &= k_3 \cdot i_h \cdot H_3 \cdot 1
\end{align*} \]

\[ \begin{align*}
q_1 &= k_1 \cdot i_h \cdot H_1 \\
q_2 &= k_2 \cdot i_h \cdot H_2 \\
q_3 &= k_3 \cdot i_h \cdot H_3
\end{align*} \]

Flow Through Heterogeneous Media

Effective Permeability in Stratified Soils

\[ Q = k_{eq\text{-}vert} \cdot i_h \cdot A \]

\[ i_h \cdot H = i_{h1} \cdot H_1 + i_{h2} \cdot H_2 + i_{h3} \cdot H_3 \]

\[ \begin{align*}
\frac{Q}{k_{eq\text{-}vert} \cdot A} \cdot H &= \frac{Q}{k_1 \cdot A} \cdot H_1 + \frac{Q}{k_2 \cdot A} \cdot H_2 + \frac{Q}{k_3 \cdot A} \cdot H_3 \\
k_{eq\text{-}vert} &= \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3}}
\end{align*} \]
Flow Through
Heterogeneous Media

Effective Permeability in Stratified Soils

\[ Q = k_{eq-vert} \cdot i_h \cdot A \]
\[ = k_1 \cdot i_{h1} \cdot A = k_2 \cdot i_{h2} \cdot A = k_3 \cdot i_{h3} \cdot A \]
\[ i_h \cdot H = i_{h1} \cdot H_1 + i_{h2} \cdot H_2 + i_{h3} \cdot H_3 \]
\[ \frac{Q}{k_{eq-vert} \cdot A} \cdot H = \frac{Q}{k_1 \cdot A} \cdot H_1 + \frac{Q}{k_2 \cdot A} \cdot H_2 + \frac{Q}{k_3 \cdot A} \cdot H_3 \]

\[ k_{eq-vert} = \frac{H}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}} \]

Flow Through
Heterogeneous Media

Zones of different hydraulic conductivities

\[ \Delta q = k_1 \cdot \frac{\Delta h}{l_1} \cdot (b_1 \cdot 1) = k_2 \cdot \frac{\Delta h}{l_2} \cdot (b_2 \cdot 1) \]
\[ \frac{k_1}{k_2} = \frac{b_2}{b_1} \cdot \frac{l_1}{l_2} \]

(Das 1983)
Flow Through Heterogeneous Media

Zones of different hydraulic conductivities

\[ l_1 = AB \cdot \sin(\theta_1) = AB \cdot \cos(\alpha_1) \]
\[ l_2 = AB \cdot \sin(\theta_2) = AB \cdot \cos(\alpha_2) \]
\[ b_1 = AC \cdot \cos(\theta_1) = AC \cdot \sin(\alpha_1) \]
\[ b_2 = AC \cdot \cos(\theta_2) = AC \cdot \sin(\alpha_2) \]

then

\[ \frac{b_1}{l_1} = \frac{\cos(\theta_1)}{\sin(\theta_1)} = \frac{\sin(\alpha_1)}{\cos(\alpha_1)} = \frac{1}{\tan(\theta_1)} = \tan(\alpha_1) \]
\[ \frac{b_2}{l_2} = \frac{\cos(\theta_2)}{\sin(\theta_2)} = \frac{\sin(\alpha_2)}{\cos(\alpha_2)} = \frac{1}{\tan(\theta_2)} = \tan(\alpha_2) \]

Flow Through Heterogeneous Media

Zones of different hydraulic conductivities

\[ \frac{k_1}{k_2} = \frac{\tan(\theta_1)}{\tan(\theta_2)} = \frac{\tan(\alpha_2)}{\tan(\alpha_1)} \]

Examples:

(Das 1983)
Flow Through Heterogeneous Media

- Zones of different hydraulic conductivities

Flow Net Example:

![Flow Net Diagram](Das 1983)

Flow Through Anisotropic Media

- Anisotropic Media $k_x \neq k_y$

Laplace's equation:

$$0 = k_x \frac{\partial^2 h_T}{\partial x^2} + k_y \frac{\partial^2 h_T}{\partial y^2}$$

The equation may be easily solved by transforming coordinates from
The $(x,y)$ space to the $(x',y)$ space

where:

$$x' = \alpha \cdot x = \frac{k_y}{k_x} \cdot x$$

and:

$$x = \frac{x'}{\alpha} \Rightarrow \frac{\partial}{\partial x} = \frac{\partial}{\partial \left(\frac{x'}{\alpha}\right)}$$

$$\frac{\partial h_T}{\partial x} = \alpha \cdot \frac{\partial h_T}{\partial x'} \Rightarrow \frac{\partial^2 h_T}{\partial x^2} = \alpha^2 \cdot \frac{\partial^2 h_T}{\partial x'^2}$$
Flow Through Anisotropic Media

\[ k_x \neq k_y \]

Replacing into Laplace's equation:

\[
0 = k_x \alpha \cdot \frac{\partial^2 h_T}{\partial x^2} + k_y \cdot \frac{\partial^2 h_T}{\partial y^2}
\]

And that is:

\[
0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2}
\]

The problem may then be solved using flow nets by redrawing the cross-section using \( x' = x \cdot \sqrt{\frac{k_y}{k_x}} \)...

Flow Through Anisotropic Media

\[ k_x \neq k_y \]

The flow in an anisotropic media:

\[
Q = \alpha \cdot x \cdot k_{eq} \cdot \frac{\partial h_T}{\partial y} = x \cdot k_y \cdot \frac{\partial h_T}{\partial y}
\]

\[
\alpha \cdot k_{eq} = k_y \Rightarrow \sqrt{\frac{k_y}{k_{eq}}} \cdot k_{eq} = k_y
\]

Transformed permeability:

\[ k_{eq} = \sqrt{k_x \cdot k_y} \]
Coupled Flow

- Coupled Phenomena
  It is justified by:
  - Le Chatelier's Principle & Energy Conservation
  - Example: Electrosmosis

![Diagram](Das 1983)

Seepage Control - Filters

- Seepage: Cut-off walls
  - Impervious blankets

- Erosion and piping: Filters
  Requirements
  - Piping: $D_{15(\text{filter})} \leq 5 \ D_{85 (\text{soil})}$
  - Permeability: $D_{15(\text{filter})} \geq 5 \ D_{15 (\text{soil})}$
  - Uniformity: $D_{50(\text{filter})} \leq 25 \ D_{50 (\text{soil})}$
Bibliography